

Implementation of many-qubit Grover search by cavity QED

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We explore the possibility of N -qubit ($N > 3$) Grover search in cavity QED, based on a fast operation of N -qubit controlled phase-flip with atoms in resonance with the cavity mode. We demonstrate both analytically and numerically that, our scheme could be achieved efficiently to find a marked state with high fidelity and high success probability. As cavity decay is involved in our quantum trajectory treatment, we could analytically understand the implementation of a Grover search subject to dissipation, which would be very helpful for relevant experiments.

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1. INTRODUCTION

Cavity quantum electrodynamics (QED) has got much progress over past several years [1]. People have achieved various optical and microwave cavities, with which some remarkable quantum phenomena have been demonstrated, such as strong coupling of a single atom with a single photon [2], quantum logic gating between two atomic qubits [3], photonic blocking [4], efficient generation of single photons [5], and so on.

In the present work, we will investigate how to realize a Grover search with more than three qubits by cavity QED. The Grover search, as one of the most frequently mentioned quantum algorithms, could effectively exemplify the potential speed-up offered by quantum computers [6]. Recently, many authors have dedicated themselves to the search algorithm by adiabatic evolution methods [7, 8, 9] or nonadiabatic scheme [10]. Although achievement of these schemes needs stringent conditions and demanding techniques [11], they are really wonderful ideas. On the other hand, some authors had addressed the effect of decoherence [12], noise [13], and gate imperfection or errors [14] on the efficiency of quantum algorithms. We also noticed that, there had been intensive interests in achieving Grover search algorithm theoretically and experimentally by using NMR systems [15], trapped ions [16, 17, 18], linear optical elements [19, 20], cavity QED [21, 22, 23, 24, 25], and superconducting meso-circuits [26].

Our interest is in the experimental feasibility of Grover search by a microwave cavity. A recent work [23], with two of the authors joined in, showed a three-qubit Grover search implementation in a microwave cavity. As it makes use of the resonant interaction of the atoms with the cavity mode and considers the detrimental influence from the cavity decay, the implementation of [23] is close to the reach of current technique. However, that scheme is hard to be extended to the case with more than three qubits due to the method [27] used there. In the present work, we will show that, after some modification of the method in [27], we could accomplish a high-fidelity conditional phase flip (CPF) for many atomic qubits flying through a microwave cavity, based on which an N -qubit ($N > 3$) Grover search might be achieved in a straightforward way. The main idea is that, we make a smart qubit encoding on the atomic levels, and send the atoms through a microwave cavity for resonant interaction. As long as we could exactly control the interaction time and the trajectories of the flying atoms, we may accomplish the N -qubit CPF gate by one step of implementation, based on which we could efficiently carry out an N -qubit Grover search with high fidelity. However, as the Grover search with more than two qubits works only probabilistically and also as our one-step implementation of many-qubit CPF gate intrinsically owns imperfection, the situation in our case is much more complicated than in two-qubit case. To make our scheme more realistic, we involve the cavity decay in our treatment, and as done in [23], we will try to present some analytical expressions for the state evolution during our implementation. As a result, we could explicitly assess how well a Grover search is made subject to these disadvantageous factors.

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2. N-QUBIT CPF GATE SUBJECT TO CAVITY DECAY

As described in [23, 27], we consider the resonant interaction of N three-level atoms with a single-mode cavity. As shown in Fig. 1, the three levels can be described by $|i_j\rangle$, $|g_j\rangle$, and $|e_j\rangle$, in which the subscript j means the j th atom, the states $|e_j\rangle$ are excited states and $|i_j\rangle$, $|g_j\rangle$ are ground states. As $|g_j\rangle$ and $|e_j\rangle$ are coupled in resonance by the cavity mode, the states $|i_j\rangle$ are not involved in the interaction with the cavity mode throughout our treatment. We encode the first qubit $|0\rangle_1$ ($|1\rangle_1$) on $|e_1\rangle$ ($|g_1\rangle$), and other qubits $|0\rangle_k$ ($|1\rangle_k$) on $|i_k\rangle$ ($|g_k\rangle$) with $k = 2, 3, \dots, N$. Considering the cavity with weak decay, which implies no photon actually leaking out of the cavity during our implementation, we may write the Hamiltonian in units of $\hbar = 1$ as,

$$H = \sum_{i=1}^N \Omega_i (a^\dagger \sigma_i^- + a \sigma_i^+) - i \frac{\kappa}{2} a^\dagger a, \quad (1)$$

where Ω_i is the coupling constant of the i th atom to the cavity mode, and the atomic spin operators for raising and lowering are $\sigma_i^+ = |e_i\rangle \langle g_i|$ and $\sigma_i^- = |g_i\rangle \langle e_i|$, respectively. a^\dagger (a) is the creation (annihilation) operator of the cavity mode. We will first consider an implementation of N -qubit CPF under cavity dissipation, based on which an N -qubit Grover search would be achieved in the next section.

Assuming the initial atomic state in $|e_1\rangle \prod_{l,j=2,l \neq j}^N |g_l\rangle |i_j\rangle$ and the cavity initially in the vacuum state $|0\rangle$, we could deduce the evolved state by directly solving the Schrödinger equation regarding Eq. (1), which yields [27],

$$\begin{aligned} |e_1\rangle \prod_{l,j=2,l \neq j}^N |g_l\rangle |i_j\rangle |0\rangle &\longrightarrow \left[\frac{\Omega_1^2}{G_m^2} \exp(-\kappa t/4) \left(\cos(A_{m+1,\kappa} t) + \frac{\kappa}{4A_{m+1,\kappa}} \sin(A_{m+1,\kappa} t) \right) + \frac{G_m^2 - \Omega_1^2}{G_m^2} \right] \times \\ &|e_1\rangle \prod_{l,j=2,l \neq j}^N |g_l\rangle |i_j\rangle |0\rangle + \frac{\Omega_1^2}{G_m^2} \left[\exp(-\kappa t/4) \left(\cos(A_{m+1,\kappa} t) + \frac{\kappa}{4A_{m+1,\kappa}} \sin(A_{m+1,\kappa} t) \right) - 1 \right] \times \\ &|g_1\rangle \sum_{k=2}^{m+1} \Omega_k |e_k\rangle \prod_{l,j=2,l \neq j \neq k}^N |g_l\rangle |i_j\rangle |0\rangle - (i\Omega_1/A_{m+1,\kappa}) \sin(A_{m+1,\kappa} t) |g_1\rangle \prod_{l,j=2,l \neq j}^N |g_l\rangle |i_j\rangle |1\rangle, \end{aligned} \quad (2)$$

where $G_m = \sqrt{\sum_{j=1}^{m+1} \Omega_j^2}$ and $A_{m+1,\kappa} = \sqrt{\sum_{j=1}^{m+1} \Omega_j^2 - \kappa^2/16}$, with m the number of the atoms initially in the ground state $|g\rangle$.

In order to accomplish an N -qubit CPF gate, we should also pay attention to the evolution with the initial states $|e_1\rangle \prod_{j=2}^N |i_j\rangle |0\rangle$, which is

$$\begin{aligned} |e_1\rangle \prod_{j=2}^N |i_j\rangle |0\rangle &\longrightarrow \left[\exp(-\kappa t/4) \left(\cos(A_{1,\kappa} t) + \frac{\kappa}{4A_{1,\kappa}} \sin(A_{1,\kappa} t) \right) \right] \times \\ &|e_1\rangle \prod_{j=2}^N |i_j\rangle |0\rangle - (i\Omega_1/A_{1,\kappa}) \sin(A_{1,\kappa} t) |g_1\rangle \prod_{j=2}^n |i_j\rangle |1\rangle, \end{aligned} \quad (3)$$

with $A_{1,\kappa} = \sqrt{\Omega_1^2 - \kappa^2/16}$. Assuming the coupling constant satisfies the condition $\Omega = \Omega_2 = \Omega_3 = \dots = \Omega_N \gg \Omega_1$, we have, under the approximation of short time evolution with $\sin(A_{1,\kappa} t) \ll \cos(A_{1,\kappa} t)$, Eq. (3) reduced to

$$|e_1\rangle \prod_{j=2}^N |i_j\rangle |0\rangle \longrightarrow \alpha |e_1\rangle \prod_{j=2}^N |i_j\rangle |0\rangle, \quad (4)$$

with $\alpha = \left[\exp(-\kappa t/4) \left(\cos(A_{1,\kappa} t) + \frac{\kappa}{4A_{1,\kappa}} \sin(A_{1,\kappa} t) \right) \right]$. Under the same condition as above, Eq. (2) similarly reduces to

$$|e_1\rangle \prod_{\substack{j,k=2, \\ j \neq k}}^N |g_k\rangle |i_j\rangle |0\rangle \longrightarrow \beta_m |e_1\rangle \prod_{\substack{j,k=2, \\ j \neq k}}^N |g_k\rangle |i_j\rangle |0\rangle, \quad (5)$$

with $\beta_m = \left[\frac{\Omega_m^2}{G_m^2} \exp(-\kappa t/4) \left(\cos(A_{(m+1)\kappa} t) + \frac{\kappa}{4A_{(m+1)\kappa}} \sin(A_{(m+1)\kappa} t) \right) + \frac{G_m^2 - \Omega_m^2}{G_m^2} \right]$. In order to make a CPF gating, we need $\alpha \rightarrow -1$ in Eq. (4), which corresponds to $t = \pi/\sqrt{\Omega_1^2 - \kappa^2/16}$. As a result, the coefficient β_m becomes

$$\beta_m = \frac{-\alpha}{1 + m\eta^2} (\cos \vartheta + \mu \sin \vartheta / \sqrt{16 + 16m\eta^2 - \mu^2}) + \frac{m\eta^2}{1 + m\eta^2}, \quad (6)$$

where $\vartheta = \pi\sqrt{1 + 16m\eta^2/(16 - \mu^2)}$, $\eta = \Omega/\Omega_1$ and $\alpha = -\exp(-\pi\mu/\sqrt{16 - \mu^2})$ with $\mu = \kappa/\Omega_1$. It is easily verified that $\alpha \rightarrow -1$ and $\beta_m \rightarrow 1$ in the case of $\kappa \rightarrow 0$. We may take $N = 4$ as an example to show the CPF more explicitly. Following the deduction above, we have the four-qubit CPF written as $U^{(4)} = J_{0000} = \text{diag}\{\alpha, \beta_1, \beta_1, \beta_1, \beta_2, \beta_2, \beta_2, \beta_3, 1, 1, 1, 1, 1, 1, 1, 1\}$ with respect to the bases $|e_1 i_2 i_3 i_4\rangle, |e_1 i_2 i_3 g_4\rangle, |e_1 i_2 g_3 i_4\rangle, |e_1 g_2 i_3 i_4\rangle, |e_1 i_2 g_3 g_4\rangle, |e_1 g_2 i_3 g_4\rangle, |e_1 g_2 g_3 i_4\rangle, |e_1 g_2 g_3 g_4\rangle, |g_1 i_2 i_3 i_4\rangle, |g_1 i_2 i_3 g_4\rangle, |g_1 i_2 g_3 i_4\rangle, |g_1 g_2 i_3 i_4\rangle, |g_1 i_2 g_3 g_4\rangle, |g_1 g_2 i_3 g_4\rangle, |g_1 g_2 g_3 i_4\rangle$, and $|g_1 g_2 g_3 g_4\rangle$, respectively, where the latter eight states do not evolve under the Hamiltonian of Eq. (1) in the case of vacuum cavity state. To assess the CPF gating, we assume the atoms to be initially in $|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle + |e_1\rangle)(|g_2\rangle + |i_2\rangle) \bullet \bullet \bullet (|g_N\rangle + |i_N\rangle)$. We may calculate the fidelity and the success probability by employing the relations $F = \langle \Psi_f | U^{(n)\dagger} | \Psi_0 \rangle \langle \Psi_0 | U^{(n)} | \Psi_f \rangle$ [28] and $P = \langle \Psi_f | \Psi_f \rangle$ [23, 27]. As shown in Fig. 2, we could reach considerably high fidelity F and high probability of success P as long as $\eta \rightarrow 10$, even in the case of relatively large decay rate. In what follows, we will employ our CPF to carry out Grover search under weak dissipation.

3. N-QUBIT GROVER SEARCH

Generally speaking, provided that initial state of the system has been prepared in an average superposition state $|\Psi_0\rangle = (1/\sqrt{2^N}) \sum_{i=0}^{2^N} |i\rangle$, Grover search can be depicted as the iterative operation $\hat{D}^{(N)} J_\tau$ by at least $\pi\sqrt{2^N}/4$ times for finding the marked state $|\tau\rangle$ with an optimal probability, where the quantum phase gate $J_\tau = I - 2|\tau\rangle\langle\tau|$ (with I being the identity matrix) plays an important role to invert the amplitude of the marked state, and the diffusion transform $\hat{D}^{(N)}$ is defined as $\hat{D}_{ij} = 2/K - \delta_{ij}$ ($i, j = 1, 2, \dots, K, K = 2^N$). Considering the qubit subspace spanned by $\{|e_1\rangle, |g_1\rangle, |i_2\rangle, |g_2\rangle \dots |i_{N-1}\rangle, |g_{N-1}\rangle, |i_N\rangle, |g_N\rangle\}$, to make a Grover search, we could construct following transformation,

$$Q^{(N)} = W^{\otimes N} J_{00\dots 0}^{(N)} W^{\otimes N} J_\rho = W^{\otimes N} J_{e_1 i_2 \dots i_n}^{(N)} W^{\otimes N} J_\rho = -\hat{D}^{(N)} J_\rho, \quad (7)$$

where $J_{00\dots 0}^{(N)} = \text{diag}\{-1, 1, \dots, 1\} = I^{(N)} - 2|00\dots 0\rangle\langle 00\dots 0|$ (with superscript (N) denoting N -qubit case and \dots denoting N elements) and the Hadamard gate in our computational subspace is given by

$$W^{\otimes N} = \prod_{i=1}^N W_i = \left(\frac{1}{\sqrt{2}}\right)^N \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \dots \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (8)$$

In the case of N qubits, Eq. (7) implies that the diffusion transform $\hat{D}^{(N)} = -W^{\otimes N} J_{e_1 i_2 \dots i_n}^{(N)} W^{\otimes N}$ is always unchanged no matter which state is to be searched. The only change is the operator J_ρ for different marked states. Based on the CPF gate constructed in last section, we have $J_{00\dots 0}^{(N)} \approx \text{diag}\{-1, 1, \dots, 1\} = U_{CPF}^{(N)}$, from which we could achieve the gate $J_{11\dots 1}^{(N)}$ as,

$$J_{11\dots 1}^{(N)} = S_{x,N} S_{x,N-1} \dots S_{x,1} J_{00\dots 0}^{(N)} \sigma_{x,1} \dots S_{x,n-1} S_{x,n}, \quad (9)$$

where $S_{x,j} = |i_j\rangle\langle g_j| + |g_j\rangle\langle i_j|$ with $j \neq 1$, and $\sigma_{x,1} = |e_1\rangle\langle g_1| + |g_1\rangle\langle e_1|$. To achieve $Q^{(N)}$, we will construct the CPF gate $J_\rho = I - 2|\rho\rangle\langle\rho|$. For example, in the case of $N = 10$, the number of the involved quantum states is 2^{10} , and the operation is to label a marked state by J_ρ with ρ one of the states from $\{|0000000000\rangle, |0000000001\rangle, |0000000010\rangle, \dots, |1111111111\rangle\}$. To carry out the ten-qubit Grover search, we need two ten-qubit Hadamard gates $W^{\otimes 10}$. Based on the CPF gate marking the state $|g_1 g_2 g_3 g_4 g_5 g_6 g_7 g_8 g_9 g_{10}\rangle$, we could construct other $2^{10} - 1$ gates for the marking job by single-qubit rotations. For example,

$$J_{g_1 g_2 g_3 g_4 g_5 g_6 g_7 g_8 g_9 g_{10}} = J_{1111111111} = S_{x,10} S_{x,9} S_{x,8} S_{x,7} S_{x,6} S_{x,5} S_{x,4} S_{x,3} S_{x,2} \sigma_{x,1} J_{0000000000} \sigma_{x,1} S_{x,2} S_{x,3} S_{x,4} S_{x,5} S_{x,6} S_{x,7} S_{x,8} S_{x,9} S_{x,10}. \quad (10)$$

So with a state marked, and with the ten-qubit diffusion transform $\hat{D}^{(10)}$ which is generated by combining two Hadamard gates $W^{\otimes 10}$ with $J_{0000000000}$, a full Grover search for ten qubits is available.

Let us consider a ten-qubit Grover search for the marked state $|e_1 i_2 g_3 g_4 i_5 i_6 i_7 i_8 i_9 i_{10}\rangle$ as an example. The Rydberg atoms from the 1st to the 10th are input, as shown in Fig. 1, with principal quantum numbers 49, 50 and 51 denoting $|g_j\rangle$, $|i_j\rangle$ and $|e_j\rangle$, respectively. Suppose the atoms initially prepared in a superposition $|\Psi_0\rangle = 2^{-5} \times (|e_1\rangle + |g_1\rangle)(|i_2\rangle + |g_2\rangle)(|i_3\rangle + |g_3\rangle) \cdots (|i_{10}\rangle + |g_{10}\rangle)$. We plot in Fig. 3 for the success rate of the Grover search in different cases. As the Grover search involving more than two qubits is intrinsically probabilistic, how to obtain an optimal search is a problem of interest. In contrast to the previous study for an ideal Grover search [29], our analytical expressions along with some numerics could identify the optimal search under cavity decay and other imperfection due to gating. We know from Fig. 3 that, because of the cavity decay, the more iteration steps, the more detrimental effect from the cavity decay involved. As a result, we prefer to have less iteration steps for an optimal search. The detrimental effect from the cavity decay is also reflected in the figure that the maximum value of P is slightly lower in the case of larger decay rate.

4. DISCUSSION

Our CPF gating plays the essential role in our Grover search scheme, which much reduces the implementation time compared to that by a series of two-qubit conditional gates and single-qubit gates [30]. Sec II has shown that the CPF gating time only depends on the weakest coupling Ω_1 and the cavity decay rate κ , but irrelevant to the qubit number. So our scheme could keep constant implementation time no matter how many atoms are involved, which favors a large-scale QIP. As a result, if we suppose the single-qubit rotation takes negligible time compared to the CPF gating, then our Grover search could be carried out by a constant time irrelevant to the qubit number. Specifically, assuming $\Omega = 2\pi \times 49$ kHz [1], $\Omega_1 = 2\pi \times 4.9$ kHz, and $\kappa = 0.1\Omega_1$, we have $t_0 = \pi/\sqrt{\Omega_1^2 - \kappa^2/16} \approx 102$ μ s, which is much shorter than either the cavity decay time, i.e., $2\pi/\kappa \approx 2$ ms, or the Rydberg atomic lifetime 30 ms [1]. However, for a Grover search, the operation $Q^{(N)}$ has to be made for several times depending on the specific search task. In the case of $N = 10$, the total required time for a Grover search would be about $2[\pi\sqrt{2^N}/4] \times t_0 \approx 5$ ms which is comparable to the cavity decay time.

As shown in Fig.3, our numerics shows that the iteration steps for optimal search are 25, 23, 17, respectively, in the case of $\mu = 0, 0.05$, and 0.1 . Since the operation time $t_0 = \pi/\sqrt{\Omega_1^2 - \kappa^2/16}$ varies very little in the case of weak cavity dissipation with respect to the ideal case, the less iteration steps under dissipation would definitely lead to reduction of our implementation time. Straightforward calculation shows that the required time for an optimal search are 5 ms, 4.6 ms, and 3.4 ms, respectively, in the case of $\mu = 0, 0.05$, and 0.1 . It is of importance to have a fast implementation of Grover search in view of decoherence. But if we seriously consider the values in above calculation, we could find that $\mu = 0.05$, and 0.1 correspond to dissipation time of 4 ms and 2 ms, respectively. As a result, to have a fast implementation with our scheme, we must increase the coupling strengths Ω and Ω_1 by at least ten times, which makes sure the required implementation time to be much shorter than the dissipation time.

In current microwave cavity experiments [1], we may consider the interaction between atoms and the cavity mode by $\Omega \cos(2\pi z/\lambda_0) \exp(-r^2/w^2) \sim \Omega \cos(2\pi z/\lambda_0)$ [1], where Ω is the coupling strength at the cavity center, r is the distance of the atom away from the cavity center, and λ_0 and w are the wavelength and the waist of the cavity mode, respectively. To meet the condition of $\Omega = 10\Omega_1$, we may send the first atom going through the cavity along y axis deviating from the nodes by $(\lambda_0/2\pi) \arccos(1/10)$, but other atoms through the antinodes. Specifically, for the N input atoms (suppose N being an even number for simplicity), we should have the tracks of the atomic movement as $z_1 = -(N/2)\lambda_0 + (\lambda_0/2\pi) \arccos(1/10)$, $z_2 = -(N/2 - 1)\lambda_0$, ..., $z_N = (N/2)\lambda_0$. Moreover, with current cavity QED techniques, the design in Fig. 1 could be realized by 50 separate microwave cavities with each Ramsey zone located by a cavity. Since each microwave cavity is employed to carry out a ten-qubit phase gate $J_{00\dots 0}^{(10)}$, the cavities should be identical. While searching for different states, we employ different single-qubit operations. So the Ramsey zones should be long enough to finish the necessary single-qubit operations.

However, it is highly challenging with current experimental technology to simultaneously send many Rydberg atoms through a cavity with precise tracks and velocities although two Rydberg atoms going through a microwave cavity simultaneously have been achieved [21]. Here we assess the infidelity due to the imperfection in atomic velocity by a simple example below. Consider the simple situation that the first atom moves a little bit slower in the cavity than other atoms, that is, $t_1 = t_0 + \delta t$, $t_2 = t_3 = \cdots = t_n = t_0$, with $t_0 = \pi/\sqrt{g_1^2 - \kappa^2/16}$. As shown in Fig.4 with $\eta = 10$, we can see that the infidelity increases with both δt and μ . In actual experiment, the situation is much more complicated. Besides the diversity of atomic velocity, there will be other imperfections such as classical pulse imperfection, differences of the cavities, deflected atomic trajectories and so on. Our simple estimate could show the importance of highly controllable movement of the atoms in implementing our scheme.

5. CONCLUSION AND ACKNOWLEDGEMENTS

In conclusion, we have studied many-qubit CPF gate in cavity QED under weak cavity dissipation, and explored the possibility of many-qubit Grover search in a dissipative cavity QED system. As a theoretical work, like previous publication in cavity QED, e.g., [31], we have simplified the complexity in the realistic system, and tried to present some analytical expressions for the state evolution subject to the cavity decay. Although we took an example involving ten qubits, our approach could also be easily used for three or four atomic qubits in cavity QED, which might be closer to experimental realization. On the other hand, we have noticed experimentally achieved entanglement of six photons [32], eight trapped ions [33] and twelve NMR ensemble qubits [34]. So we also hope our idea could be extended to other systems, such as to trapped ions. Furthermore, from above discussion, we have known the experimental difficulty for ten atomic qubits to be entangled and used for Grover search by current cavity QED technique. Nevertheless, we argue that our study would be useful for future QIP experiments by microwave cavity.

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Captions of Figures:

Fig. 1. Schematic setup for finding the marked state $|e_1i_2g_3g_4i_5i_6i_7i_8i_9i_{10}\rangle$ in a ten-qubit Grover search, where ten atoms are simultaneously sent through the cavity with proper speed. The ten state-selective field ionization detectors D_1, \dots, D_{10} are settled at the end of the passage for checking the states of the qubits. The operations $U^{(10)}$, $H^{\otimes 10}$, $S_{x,3}$, and $S_{x,4}$ are defined in the text. The inset shows the atomic level configuration, with bold lines for the states encoding qubits and the arrows for the coupling between the atoms and the standing wave of the cavity.

Fig. 2. The variations of the fidelity and probability of success with respect to both η and μ .

Fig. 3. The search probability of ten-qubit with respect to the iteration number under the cavity decay rate (a) $\mu = 0$, (b) $\mu = 0.05$, and (c) $\mu = 0.1$, respectively.

Fig. 4. Infidelity of ten-qubit phase gate versus time delay in the case of $\eta = 10$, where $\mu = 0.05$ and $\mu = 0.1$ are plotted by solid and dashed curves, respectively.





